Solution of convection-dominated problems by means of a conformal Petrov-Galerkin method

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ABSTRACT

A wide class of problems of Fluid Mechanics and Heat and Mass Transfer is governed by the combined effects of transport and diffusion. To solve these problems, the Galerkin Finite Element (FE) method can be the first used in view of its simplicity and ease of implementation. With this method the discrete system is obtained from the continuous weak formulation by selecting the same finite dimensional subspaces for the shape and test functions (as used to discretize the unknowns and the equations, respectively).

Unfortunately, the Galerkin FE method performs quite badly in the situations where transport prevails over diffusion. Therefore the solution of the advection-diffusion equation has been the object of extensive investigations in the literature. Multiscale and stabilized methods add stabilizing terms to the original weak formulation [1] while the Discontinuous Galerkin method is based on using discontinuous shape functions with the consequence that this approximation is no longer "internal" [2].

In this paper, we present a conformal Petrov-Galerkin FE method in order to solve the 2D advection-diffusion equation. By "conformal" we mean that the discrete system is obtained from the continuous weak formulation by selecting different finite dimensional subspaces for the shape and test functions without adding any stabilizing term. Our approach is based on classes of continuous test functions that provide exact nodal values for selected classes of solutions [3]. Combining appropriately these test functions in the general case induces a stabilizing upwinding effect which removes the wiggles obtained with the pure Galerkin method. These classes of test functions include the constant functions in order to ensure global and local conservation properties. The results obtained are of very high quality.

References

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